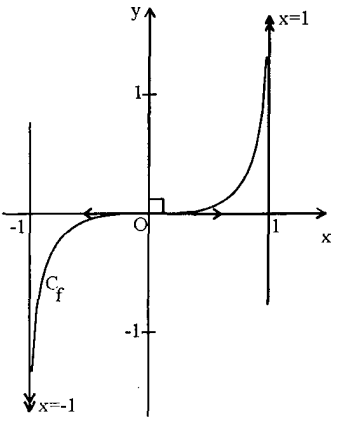
• $\lim\limits_{-1^+} f = \lim\limits_{x\to -1^+}\left[-x+\frac{1}{2}\text{Log}\frac{1+x}{1-x}\right] = -\infty$

• $\lim\limits_{1^-} f = \lim\limits_{x\to 1^-}\left[-x+\frac{1}{2}\text{Log}\frac{1+x}{1-x}\right] = +\infty$

D'où le tableau de variation de f

x	-1		0		1
f'(x)	‖	+	0	+	‖
f (x)	‖ -∞	↗	+∞	↗	‖



3/a- Soit $k \in IN^*$. $g : IR_+^* \to IR; x \mapsto g(x)=\text{Log}x - \left(\frac{x}{k} - 1 + \text{Log}k\right)$

g est dérivable sur IR_+^* et $\forall x \in IR_+^*, g'(x) = \frac{1}{x} - \frac{1}{k} = \frac{k-x}{kx}$

D'où le tableau de variation de g sur IR_+^*

x	0		k		+∞
g'(x)	‖	+	0	-	‖
g(x)	‖	↗	0	↘	‖

$\Rightarrow$ 0 est la valeur maximale de g par suite $\forall x \in IR_+^*, g(x) \le 0$

signifie que $\forall x \in IR_+^*, \text{Log}x \le \frac{x}{k} - 1 + \text{Log}k$

b- $\forall k \in IN^*$ on a $k - \frac{1}{2} < k + \frac{1}{2}$

D'où $\forall x \in \left[k-\frac{1}{2}; k+\frac{1}{2}\right]; \text{Log}x \le \frac{x}{k} - 1 + \text{Log}k$

Par suite $\int_{k-½}^{k+½} \text{Log}x.\,dx \le \int_{k-½}^{k+½}\left(\frac{x}{k} - 1 + \text{Log}k\right)dx$

$\Leftrightarrow \int_{k-½}^{k+½} \text{Log}x.\,dx \le \left[\frac{x^2}{2k} - x + (\text{Log}k)x\right]_{k-½}^{k+½} = \text{Log}k$

★$\forall k \in IN^*$ on a $\int_{k-½}^{k+½} \text{Log}x.\,dx \le \text{Log}k$

D'où $\forall n \in IN^*; \sum_{k=1}^{k=n}\int_{k-½}^{k+½} \text{Log}x.\,dx \le \sum_{k=1}^{k=n} \text{Log}k.$

★ $\sum_{k=1}^{k=n} \text{Log}k = \text{Log}1 + \text{Log}2 + \text{Log}3 + \ldots + \text{Log}n$

$= \text{Log}[1 \times 2 \times 3 \times \ldots \times n] = \text{Log}(n!).$

★ $\sum_{k=1}^{k=n}\int_{k-½}^{k+½} \text{Log}x\,dx = \int_{1-½}^{1+½} \text{Log}x\,dx + \int_{1+½}^{2+½} \text{Log}x\,dx + \ldots + \int_{n-½}^{n+½} \text{Log}x.\,dx$